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Aug. 17, 1995

Dr. Arje Nachman
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Dear Arje:

Enclosed is the final technical report of our work done under the AFOSR grant F49620-92-J-0200. I hope you will find these results interesting, and agree that our work under your grant has produced a significant impact and should be continued.

If you have any questions or comments, please feel free to call me.

Sincerely

Fazle

Fazle Hussain

Final Technical Report for AFOSR

a. Contract Information

i) Title: Vortex Core Dynamics, Complex Helical Wave Decomposition, Organization of fine-scale Turbulence and other Related Theoretical/Numerical Studies

ii) Principal Investigator: Fazle Hussain, University of Houston

iii) Contract Number: F49620-92-J-0200

iv) Program monitor: Dr. Arje Nachman

v) Period: June 1992 to May 1995

b. Description of Research

Key contributions: Several important results not only of conceptual significance but also relevant to technological applications have been obtained. These relate to theoretical and numerical studies of vortex dynamics and turbulence modeling. We showed that vortex lines in coherent structures in turbulent flows are usually helical *(i.e. polarized)*, contrary to the customary assumption of unpolarized vortex lines. More surprisingly, we find that all *coherent structures* in turbulent flows are partially polarized and are either predominantly *right-handed* or *left-handed*. The role of polarization in significantly altering the evolutionary dynamics of isolated and interacting vortices was demonstrated by analyzing axisymmetric *vortex rings with swirl*. Polarized axisymmetric vortex dynamics are easily explained using a new *topology-based* analysis approach which also yields a rigorous definition of *vortex reconnection*. Furthermore, the critical role played by polarized vortex dynamics is illustrated by the *core dynamics* (involving vorticity wavepacket propagation along the vortex axis caused by helical vortex lines) which is a dominant and heretofore unknown *transition mechanism* in mixing layers; in fact, core dynamics induces transition at lower perturbation amplitudes than the much studied Corcos-Lin mechanism and the rib-roll interactions, and it appears to be a *generic transition mechanism* in all free shear flows. We have also explained the mechanisms of *direct coupling* between coherent structures and fine-scale turbulence, which reveals the failure of the gradient transport hypothesis and questions the validity of Kolmogorov's theory of local isotropy. A rigorous, objective *definition of a*

vortex was developed to identify dynamically significant vortical structures in both free and wall-bounded turbulent flows. This definition was used to educe near-wall structures in a turbulent channel flow, explain their dynamics and construct a *conceptual model* of structure dynamics consistent with nearly all reported experimental results. An analytical approach to incorporate structural information into *self-consistent models of turbulent flows* was developed and illustrated via a *structural model* (without any empirical input) of a turbulent mixing layer. Furthermore, a *variational principle for turbulent flows* was developed and used to predict the measured eddy viscosity in a jet.

We recognize vortex breakdown to be an *internal separation* (away from a solid boundary) and have begun explaining the structure (in the inviscid limit) and the dynamics of bubble-type *vortex breakdown*. We have analyzed swirling jets to explain and predict the experimentally observed hysteresis, vortex breakdown and vortex consolidation phenomena. Experiments in mixing layers were used to provide vortex dynamics-based explanations for the predictions of and the deviations from weakly nonlinear theories of *subharmonic resonance*. As necessary precursors for developing real-time control techniques for open flows, we have experimentally identified low-dimensional temporal dynamics (characterized by periodic and chaotic attractors) and have proposed new *spatiotemporal* measures for mixing layer dynamics.

SUMMARY OF IMPORTANT RESULTS

In the following, we summarize only the important results achieved under individual studies performed for the AFOSR contract and emphasize scientific significance whenever possible. The references to papers based on these results are included and may be referred for further details.

1. Core Dynamics on a Column Vortex

We have shown that variations in core size play a crucial role in the dynamics in 3D-vortex flows and that they in general cannot be ignored as has previously been assumed. We examined the core dynamics of an isolated axisymmetric vortex column with a nonuniform core. In this simple and idealized configuration, the effects of the core dynamics are seen most clearly because they are decoupled from other effects, such as interactions with other vortices and self-induced displacement of the axis. We have shown analytically that the core dynamics results from a distinct physical mechanism, namely differential rotation along axisymmetric vortex surfaces. Furthermore, we have shown that the core dynamics is neither pure wavemotion nor pure mass transport, but a combination of both. We have shown and explained why core dynamics is highly Re-sensitive. Moreover, we found that the core variations — which are likely in practical flow situations — do not disappear by inviscid effects. By examining viscous dissipation we found that core dynamics results in a significantly higher dissipation rate and that dissipation is the only effect that reduces core size variations. In fact, we found that the frequency of the core size oscillations increases with Re, *but* to a finite limit as Re $\rightarrow \infty$. A striking, newly observed feature resulting from core dynamics is the appearance and disappearance of low enstrophy pockets inside the vortex.

Paper 1

Melander, M.V. & Hussain, F. " Core dynamics on a vortex column." *Fluid Dyn. Res.*, **13**, pp1-37, (1994).

2. Polarized Vorticity Dynamics on a Vortex Column

We have shown that vortex core dynamics results from the interaction of two slowly deforming, but overlapping, helical vortex structures. These are the left- and righthanded components of the vortex, and are obtained by a generalized Helmholtz decomposition (the complex helical wave decomposition) of the vorticity field. This

decomposition is based on a Fourier expansion in eigenfunctions of the curl operator, which has only real eigenvalues λ. The sum of eigenmodes with $\lambda > 0$ ($\lambda < 0$) constitutes the right (left) polarized component, and the vector lines of the field (e.g. vortex lines) are locally right (left) handed helices. We find that for a localized vortex the polarized structures are also localized, a crucial result for physical space applications. The polarized vortex structures deform slowly (compared to unpolarized structures) and behave almost like solitary waves when isolated. We show that this is because the nonlinearity in the Navier-Stokes equations is largely suppressed between eigenmodes of the same polarity. Moreover, the helicity of polarized structures is very high. The interaction between overlapping polarized structures, however, gives each structure a different propagation velocity, and also results in some additional deformation. We show that the latter occurs mainly in two places, at the front of the structure where a low enstrophy bubble forms (which is a permanent feature in each of the polarized packets), and at the back where a tail develops. Otherwise, the deformation occurs on a much slower time scale compared to that for unpolarized vortices. Thus the rapid changes in the total vorticity field results from the superposition of two slowly deforming structures moving in opposite directions, as is the case for the 1D wave equation.

Paper 2

Melander, M.V. & Hussain, F. " Polarized vorticity dynamics on a vortex column. " *Phys. Fluids A.*, 5, pp1992-2003, (1993).

3. Coupling Between a Coherent Structure and Fine Scale Turbulence

Our direct numerical simulations show that a coherent structure (C) in an initially fine-scale turbulent field breeds secondary structures in its vicinity. These are organized. Shaped like concentric spiral threads perpendicular to the axis of C, each is found to be highly polarized with the azimuthal vorticity component being the dominant. The threads occur in pairs, and the polarization alternates between adjacent threads. Above a critical value (~1000) of $Re = \Gamma_C/\nu$, a small number of circulation-rich threads emerge as a result of the evolution. These secondary structures are of both practical and theoretical importance. For $Re > Re_{crit}$, the strongest threads excite bending waves on the axis of C. For $Re >> Re_{crit}$, C is eventually destroyed. We believe that this feedback phenomenon is of critical importance for the rearrangement of coherent structures and transition to turbulence in the far fields of plane, circular and elliptic jets. Turbulent mixing near C is due to entrainment and ejection of fluid by the

threads. Local isotropy assumption can not be applied near a coherent structure, because our results show anisotropy in a layer surrounding C. The threads are shown to be the combined result of three mechanisms: (l) azimuthal alignment of small-scale vorticity by the strain rate field of C; (2) merger (pairing) and axisymmetrization as in 2D turbulent flows, enabled by the alignment; (3) and polarization by differential rotation around the spiral threads.

<u>Paper 3</u>

Melander, M.V. & Hussain, F. "Coupling between a coherent structure and fine-scale turbulence." *Phys. Rev. E*, **48**, pp2669-2689, (1993).

4. Dynamics of a Polarized Vortex Ring

Our preliminary studies strongly suggest that polarized vortical structures are a common feature in most transitional and turbulent flows, and that they are generated during transition to turbulence. A study of the evolution and dynamics of helical vortical structures is therefore an obvious avenue for understanding of 3D vortical flows and their modelling. Poor understanding of the dynamics of both fully and partially polarized vortical structures suggests a detailed study of isolated polarized structures before interactions of such structures can be addressed. Thus, we examine one of the simplest case, namely an axisymmetric ring with swirl.

Polarization is inferred by the application of helical wave decomposition. We analyzed initially polarized isolated, viscous vortex rings through direct numerical simulations of the Navier-Stokes equations using a new spectral algorithm which is based on axisymmetric eigenfunctions of the curl operator, and is developed specifically for this investigation. Integral measures of the degree of polarization, such as the fractions of energy, enstrophy, and helicity associated with right-handed (or left handed) eigenfunctions, remains nearly constant during evolution, thereby suggesting that polarization is a persistent feature. However, for polarized rings, an axial vortex (tail) develops near axis, where the local ratio of right- to left-handed vorticity develops significant nonuniformities due to spatial segregation of polarized components. Reconnection can occur in rings when polarized and is clearly discerned from the evolution of axisymmetric vortex surfaces; but interestingly, the location of reconnection cannot be inferred form the vorticity magnitude. The ring propagation velocity decreases monotonically as the degree of initial polarization increases. Detailed

results reveal surprising differences among the evolutionary dynamics of polarized, partially polarized, and unpolarized rings.

<u>Paper 4</u>

Virk, D., Melander, M.V. & Hussain, F. "Dynamics of a polarized vortex ring." *J. Fluid Mech.*, **260**, pp23-55, (1994).

5. Topological Vortex Dynamics in Axisymmetric Flows

Upon examining the topology of vortex lines and surfaces in incompressible viscous axisymmetric flows with swirl, we argue that the evolving topology of the vorticity field must be examined in terms of axisymmetric vortex surfaces rather than lines, because only the surfaces enjoy structural stability. The meridional cross sections of these surfaces are the orbits of a dynamical system with the azimuthal circulation being a Hamiltonian H and with time as a bifurcation parameter μ. The dependence of H on μ is governed by the Navier-Stokes equations; their numerical solutions provide H. The level curves of H establish a time history for the motion of vortex surfaces, so that the circulation they contain remains constant. Equivalently, there exists a *virtual velocity field* in which the motion of the vortex surfaces is frozen almost everywhere; the exceptions occur at critical points in the phase portrait where the virtual velocity is singular. The separatrices emerging from saddle points partition the phase portrait into *islands;* each island corresponds to a structurally stable vortex structure. By using the flux of the meridional vorticity field, we obtain a precise definition of reconnection: *the transfer of flux between islands*. Local analysis near critical points shows that the virtual velocity (because of its singular behavior) performs 'cut-and-connect' of vortex surfaces with the correct rate of circulation transfer — thereby validating the longstanding viscous 'cut-and-connect' scenario which implicitly assumes that vortex surfaces (and vortex lines) can be followed over a short period of time in a viscous fluid. Bifurcations in the phase portrait represent (contrary to reconnection) changes in the topology of the vorticity field, where islands spontaneously appear or disappear. Often such topology changes are catastrophic, because islands emerge or perish with finite circulation. These and other phenomena were illustrated by direct numerical simulations of vortex rings at a Re of 800.

<u>Paper 5</u>

Melander, M.V. & Hussain, F. "Topological vortex dynamics in axisymmetric viscous flows." *J. Fluid Mech.*, **260**, pp57-80, (1994).

6. Interactions of Polarized Axisymmetric Vortex Rings

Axisymmetric flows with swirl possess exactly one family of closed vortex surfaces; these are toroidal. We find that these surfaces are instrumental for identifying vortical structures as well as for understanding the flow physics even in the presence of viscosity. We have derived an inviscid conservation law for material volumes enclosed by any of these surfaces. From this law follows not only that circulation and helicity are conserved, but also the azimuthal and meridional helicity are separately constant. In the presence of viscosity, "vortex islands" consisting of nested toroidal vortex surfaces were related to global bifurcation theory for dynamical systems. Here the phase plane trajectories are the instantaneous vortex lines, and time is the bifurcation parameter. Because the toroidal vortex surfaces foliate the phase space, the dynamical theory for analyzing the phase portrait is readily available. Only when bifurcations occur does the topology of toroidal vortex surfaces change. Hence, a precise and theoretically well rounded technique for following individual structures in time in a viscous flow is established. We successfully apply this technique to direct numerical simulations of close interactions, particularly pairing, of vortex rings with high helicity Moreover, we have demonstrated how and why the toroidal vortex surfaces control the dynamics of the flow.

Paper 6

Melander, M.V., Virk, D., & Hussain, F., "Toroidal vortex surfaces and axisymmetric flows with swirl." *J. Fluid Mech.*, 1995 (submitted).

7. Polarized Vortex Reconnection

As interacting vortices in the aircraft contrails and turbulent flows are likely to have an axial flow associated with them, we think that reconnection of partially polarized vortex tubes is more relevant than that of unpolarized vortex tubes studied to date. We have simulated the reconnection of antiparallel vortex tubes as well as circular colliding rings, for both the same and opposite polarizations. We have established that the axial flows critically affect the vortex reconnections and hence cascade. Further, the directions of axial flows in the interacting vortical structures also determine the final outcome. Effects of increasing the azimuthal velocity for the same azimuthal vorticity have also been explored. We find that for very low levels of azimuthal velocity after the interaction the final rings are symmetric and orthogonal to the initial—the same as in conventional studies with no axial flow. When the maximum axial and azimuthal

velocities are comparable, the final structure is quite different. For opposite directions of initial azimuthal velocities in the rings, the reconnections lead to one much larger vortex ring. While for the same directions of initial azimuthal flow the final rings are not orthogonal to the initial rings and the scales of all the structures are comparable. These results are being analyzed in detail . We plan to provide quantitative measures for the effects of azimuthal flow on vortex reconnections by employing helical wave decomposition. Also the differences in cascade in flows with helical structures and helical flows will be explored. Moreover, the details of reconnection mechanism in vortex rings with azimuthal flow will be elucidated in terms of the vortex line and vortex surface topologies. Based on these preliminary observations that cascade properties of turbulent flows can be significantly affected by introducing vortical structures with axial flow, we will study the effects of introducing swirl in jets.

Paper 7

Virk, D. & Hussain, F. "Reconnection of vortex tubes with axial flow." 1995 (in preparation).

8. Core Dynamics Instability in Mixing Layers

We have investigated a new mechanism of small-scale transition via *core dynamics instability* (CDI) in an incompressible plane mixing layer, a transition which is not reliant on the presence of longitudinal vortices ('ribs') and which can originate much earlier than rib-induced transition. Both linear stability analysis and direct numerical simulation are used to describe CDI growth and subsequent transition in terms of vortex dynamics and vortex line topology. CDI is characterized by amplifying oscillations of core size nonuniformity and meridional flow within spanwise vortices ('rolls'), produced by a coupling of roll swirl and meridional flow that is manifested by helical twisting and untwisting of roll vortex lines. We find that energetic CDI is excited by subharmonic oblique modes of shear layer instability after roll pairing, when adjacent rolls with out-of-phase undulations merge. Starting from moderate initial disturbance amplitudes, twisting of roll vortex lines generates within the paired roll opposing spanwise flows which even exceed the freestream velocity. These flows collide to form a nearly irrotational bubble surrounded by a thin vorticity sheath of a large diameter, accompanied by folding and reconnection of roll vortex lines and local transition. We find that accelerated energy transfer to high wavenumbers precedes the development of roll internal intermittency; this transfer, inferred from increased energy

at high wavenumbers and an intensification of roll vorticity, occurs prior to the development of strong opposite-signed (to the mean) spanwise vorticity and granularity of the roll vorticity distribution. We demonstrate that these core dynamics are not reliant upon special symmetries and occur as well in the presence of moderate-strength ribs, despite entrapment of ribs within pairing rolls. In fact, the roll vorticity dynamics are dominated by CDI if ribs are not sufficiently strong to first initiate transition; thus CDI may govern small-scale transition for moderate initial 3D disturbances, typical of practical situations. Results suggest that CDI constitutes a new generic mechanism for transition to turbulence in shear flows.

Paper 8

Schoppa, W., Hussain, F., & Metcalfe, R.W., "A new mechanism of small-scale transition in a plane mixing layer: core dynamics of spanwise vortices." *J. Fluid Mech.*, **298**, pp23-80, (1995).

Paper 9

Hussain, F., Metcalfe, R. & Schoppa, W. "Core dynamics instability: a new transition mechanism in a mixing layer." *Developments in Fluid Dynamics & Aerospace Engr.* (eds. Deshpande, Prabhu, Sreenivasan & Viswanath), Interline Publ. India, pp42-69 (1995).

Paper 10

Schoppa, W. , Husain, H. S. & Hussain, F. , "Nonlinear instability of free shear layers : subharmonic resonance and three dimensional vortex dynamics." in *Nonlinear Instability of Nonparallel Flows* (eds. S. P. Lin, W. R. C. Philips & D. T. Valentine) Springer Verlag pp251- 280, (1994).

9. Definition of a Vortex

Considerable confusion surrounds the longstanding question as to what constitutes a vortex, especially in a turbulent flow. This question, frequently misunderstood as academic, has recently acquired particular significance since coherent structures (CS) in turbulent flows are now commonly regarded as vortices. An objective definition of a vortex should permit the use of vortex dynamics concepts to educe CS, to explain formation and evolutionary dynamics of CS, to explore the role of CS in turbulence phenomena, and to develop viable turbulence models and control strategies for turbulence phenomena. We propose a definition of a vortex in an incompressible flow in terms of the eigenvalues of the symmetric tensor $\mathbf{S}^2 + \Omega^2$; here $\mathbf{S}$ and Ω are respectively the symmetric and antisymmetric parts of the velocity gradient tensor $\nabla \mathbf{u}$. This definition captures pressure minimum in a plane perpendicular to the vortex axis

at high Reynolds numbers, and also accurately defines vortex cores at low Reynolds numbers, unlike a pressure minimum criterion. We compare our definition with prior schemes/definitions using exact and numerical solutions of the Euler and Navier-Stokes equations for a variety of laminar and turbulent flows. In contrast to definitions based on the positive second invariant of $\nabla \mathbf{u}$ or the complex eigenvalues of $\nabla \mathbf{u}$, our definition accurately identifies the vortex core in flows where the vortex geometry is intuitively clear.

Paper 11

Jeong, J. & Hussain, F., " On the identification of a vortex. " *J. Fluid Mech.*, **285**, pp69-94, (1995).

10. Near-wall Coherent Structures

Coherent Structures (CS) near the wall in a numerically simulated turbulent channel flow were educed using a conditional sampling scheme which extracts the entire extent of vortical structures. Such structures are detected from the instantaneous flow field using our newly developed vortex definition — a region of negative λ_2, the second largest eigenvalue of the tensor $S_{ik}S_{kj} - \Omega_{ik}\Omega_{kj}$ — which accurately captures the structure details (unlike velocity, vorticity or pressure-based eduction). Extensive testing has shown that λ_2 correctly captures vortical structures, even in the presence of strong shear near the wall. We have shown that the dominant near-wall educed (i.e. ensemble averaged after proper alignment) CS are highly elongated quasi-streamwise vortices; the CS is inclined 9^0 in the vertical (x, y) plane and tilted $\pm 4^0$ in the horizontal (x, z) plane. The vortices of alternating sign overlap in x as a staggered array; there is no indication of hairpin vortices, not only in educed data but also in the instantaneous field. Our model of the CS array reproduces nearly all experimentally observed events reported in the literature, such as VITA, gradient Reynolds stress distribution, wall pressure variation, elongated low speed streaks and spanwise shear, *etc.* In particular, a phase difference (in space) between streamwise and vertical velocity fluctuations created by the CS causes Q4 events to dominate Q2 near the wall and also creates counter-gradient Reynolds stresses (such as Q1 and Q3 events) above and below the CS. We also show that these effects are accurately modeled by a half of a Batchelor's dipole, embedded in a background shear U(y). The CS tilting (in (x,z) planes) is found to be responsible for CS sustenance through redistribution of streamwise turbulent kinetic energy to normal and spanwise components through coherent pressure-strain effects.

Paper 12

Jeong, J., Hussain, F., Schoppa, W. & Kim, J. , "Coherent structures near the wall in turbulent channel flow. " *J. Fluid Mech.* 1995 (submitted).

11. Steady Internal Separation of Vortical Flows

We have addressed the classical problem of indeterminacy of steady, inviscid, incompressible vortical flows in planar and axisymmetric geometries. In a planar flow, for sufficiently large, constant inflow vorticity, internal separation is inevitable. We evaluated various approaches to determine the flow in the separation zone, proved total non-uniqueness of recirculation zones (which may be inserted in any steady flow and which can have arbitrarily large enough vorticity), and have shown that the stagnation zone is the most relevant model for internal separation. We have proved the existence and uniqueness theorems for stagnation zones and shown that kinetic energy of the entire flow domain is the least for flows with stagnation zones; this lends additional support for the stagnation zone model.

The above analysis was then extended to axisymmetric flows with swirl. A new approach to explain vortex breakdown was developed by modeling the breakdown bubble as a stagnation zone in a semi-infinite pipe flow. We have shown how the semi-infinite flow domain allows prediction of vortex breakdown to be a result of spatial instability and bifurcation for strong swirl (consistent with experiments).

We have also obtained analytical solutions to several practical problems, including flows in a square domain, in a river, and past a smoothing screen in both planar and axisymmetric configurations.

Paper 13

Goldshtik, M. & Hussain, F., "Steady inviscid planar vortical flows with stagnant separation zones." *J. Fluid Mech.* 1995 (submitted).

12. Vortex Breakdown in a Pipe Flow

Axisymmetric swirling flow in a semi-infinite circular pipe was investigated, and analytical solutions for inviscid, steady, incompressible flows were obtained. Along with the axial velocity and axial vorticity, the azimuthal vorticity at the pipe entrance is specified. Vortex breakdown was shown to occur as a result of structural instability (i.e. a drastic flow transformation associated with folds in the parameter space) due to violation of cyclostrophic balance. When the degree of swirl at the entrance exceeds

some critical value, analytic solutions reveal folds, limit points, nonuniqueness and nonexistence. When a stagnation point and a near axis separation zone appear, the well known steady solution indeterminacy arises. We consider this steady regime to be the infinite time limit of an unsteady flow and use initial conditions to overcome the indeterminacy. In particular, we show that for a flow starting from rest or for an initially potential flow, stagnation zone is the most appropriate model for the steady breakdown bubble. We suggest that this model is preferable to the conventional analytic continuation model because the latter leads to singularity and nonexistence of solutions. In the stagnation zone model, solutions always exist and, for large enough inflow swirl, have nonuniqueness and folds in the parameter space, thus predicting the experimentally observed hysteretic jump transitions.

Paper 14

Goldshtik, M. & Hussain, F., " On vortex breakdown in a pipe flow." *J. Fluid Mech.* 1995 (submitted).

13. Numerical Studies of Internal Separation (Vortex Breakdown)

Prompted by the indeterminacy of inviscid flows containing separation zones, we addressed the problem of determination of the *inviscid limit* of recirculation zones occurring away from a solid boundary (hereafter called *internal separation*) in planar and swirl-free axisymmetric flows. Unlike in the case of external separation occurring at a solid boundary, Batchelor's model is not useful to determine the inviscid limit of internal separation; thus a new approach is required to determine the vorticity field within the recirculation zone. We have shown for both planar and axisymmetric viscous flows that, unlike external separation, internal separation can remain stable even at high Reynolds numbers (Re). We examine separation zone structure at high *Re* leading to the inviscid limit. The flow is essentially inviscid inside and outside the separation zone except in a thin layer (with a sharp gradient of vorticity) at the boundary of the separation zone where viscous effect is significant. The balance of vorticity fluxes across this vorticity boundary layer is used to show how the constant vorticity region is sustained at high Re and also to determine the value of the constant vorticity within the separation zone. In a swirl-free axisymmetric flow, a theorem, derived from the full Navier-Stokes equations, determines the vorticity distribution within the separation zone: *the azimuthal vorticity must be continuous across the separation zone boundary.* This theorem removes the otherwise inevitable nonuniqueness of inviscid flow having a recirculation zone. We have demonstrated the validity of this

theorem by comparing the inviscid limit solutions with numerical simulations at high Re.

Paper 15

Jeong, J. & Hussain, F., " Steady Internal Separation at high Reynolds number." *J. Fluid Mech.* 1995 (submitted).

14. A Variational Principle for Turbulent Axisymmetric Jets

By employing the exact (Landau) solution for an axisymmetric laminar jet, we showed that the total rate of kinetic energy dissipation in the jet has a minimum at a value of the viscosity that agrees with the empirical (constant) eddy viscosity in an axisymmetric turbulent jet of the same momentum flux. In addition, some new and intriguing features were revealed for axisymmetric jets (the Landau, Squire, and Schlichting solutions) regarding the number of inflexion points in the velocity profile, mass and momentum fluxes, and the entrainment rate. We have shown that momentum flux is an adequate flow characteristic for the Landau jet, but not for others.

Paper 16

Goldshtik, M. & Hussain, F., " Some intriguing features of axisymmetric jets." *Phys. Rev. E*, 1995 (submitted).

15. Bifurcation Cascade in Diverging Flows

Our analysis of divergent instability (which is characterized by formation of azimuthal cells and is a generic feature of 3D steady axisymmetric, viscous incompressible diverging flows near a planar or conical surface) reveals bifurcation cascade [1] that corresponds to subsequent halving of length scales of the disturbance motion as in the (inverse) Feigenbaum and Richardson-Kolmogorov cascades. The initial axisymmetric flow is driven by surface stresses and is related to Marangoni convection in a half-space of viscous incompressible fluid with high Prandtl number. The bifurcation cascade causes splitting of the initial flow into $m=2^n$, $n=1, 2, ...$, radial near-surface jets separated by inflows at critical Reynolds number (defined as the mean radial velocity at the surface times the distance from the origin divided by viscosity) $Re_* = 46*4^n$ as n tends to infinity. For this analysis, we have derived boundary layer equations in a scale transformation invariant form; further details are available in [2].

Paper 17

Shtern, V. & Hussain, F., " Azimuthal instability of divergent flows. " *J. Fluid Mech.*, **256**, pp535-560, (1993).

Paper 18

Shtern, V. & Hussain, F., "Bifurcation cascade in a diverging flow." In *Nonlinear Stability of Nonparallel Flows* (eds. 5. P. Lin, W.R.C. Philips & D.T. Valentine), Springer Verlag, pp449-458, (1994).

16. Hysteresis in Swirling Jets

Building on our results on jump transitions in swirling flows, we considered swirling jets above a cone as a model of flows outside vortex suction devices and found four abrupt metamorphoses of a global flow pattern: (i) vortex breakdown transforming a consolidated near-axis swirling jet into a two-cell flow with inverse motion near the axis and an open swirling jet flowing out near a conical surface, (ii) vortex consolidation causing the opposite transformation, (iii) jump separation of the swirling flows from the cone, and (iv) jump attachment of the swirling jet to the cone. Also the general asymptotic theory for nearly inviscid conically similar swirling flows has been developed, and the paradoxical effect of a consolidated swirling jet with an "anti-rocket" flow force is found.

Paper 19

Shtern, V. & Hussain, F., "Hysteresis in a swirling jet as a model tornado." *Phys. Fluids A*, **5**, pp2183-2195, (1993).

17. Generation of Swirl in Liquid Cones

We show that rotation appears as a result of symmetry breaking in a steady meridional flow of a viscous incompressible fluid. This is the first demonstration of the laminar axisymmetric "vortex dynamo". Surface stresses cause a flow inside the conical meniscus such that it is towards the apex near the surface and away from apex near the symmetry axis; such a circulation is typical for electrosprays. We have proved that when the Reynolds number (the product of the radial surface velocity and the distance from the apex divided by the kinematic viscosity) exceeds a rather small threshold value, a bifurcation occurs resulting in the appearance of a steady (clockwise or

counterclockwise) swirling regime. This result agrees with recent observations of swirling flows inside menisci.

Paper 20

Shtern, V., Goldshtik, M. & Hussain, F., "Generation of swirl due to symmetry breaking." *Phys. Rev. E*, **49**, p2881-2886, (1994).

18. Hysteresis in Swirling Jets

To explain hysteretic transitions in swirling jets and to model external flows of vortex suction devices, the steady rotationally symmetric motion of a viscous incompressible fluid above an infinite conical stream surface of half -angle θ_c was studied. The flows analyzed correspond to the conically similar solutions of the Navier-Stokes equations and are characterized by circulation Γ_c given at the surface and axial flow force J_1. Asymptotic analysis of and numerical calculations show that four (for $\theta_c \leq 90$) or five (for $\theta_c > 90$) solutions exist in some ranges of Γ_c and J_1. The solution branches form hysteresis loops which are related to jump transitions between various flow regimes. Four kinds of the jumps are found: (i) vortex breakdown which transforms a near axis jet into a two cell flow with reverse flow near the axis and an annular jet fanning out along surface $\theta = \theta_s < \theta_c$; (ii) vortex consolidation causing a reversal of (i); (iii) jump flow separation from surface $\theta = \theta_c$; and (iv) jump attachment of the swirling jet to the surface . As Γ_c and/or J_1 decrease, the hysteresis loops disappear through a cusp catastrophe. The physical reasons for the solution nonuniqueness are explained and the results are discussed in the context of Squire-Benjamin and Hall theories of vortex breakdown. Also two new striking effects are found : (i) there is a pressure peak of $O(\Gamma_c^2)$ inside the annular swirling jet; and (ii) a consolidated swirling jet forms with a reversed ("anti-rocket")flow force.

Paper 21

Shtern, V. & Hussain, F., "Hysteresis in Swirling Jets." *J. Fluid Mech.* 1995 (in press).

19. Subharmonic Resonance in a Shear Layer

The subharmonic resonance phenomenon was studied using hot-wire measurements and flow visualization in an initially laminar shear layer forced with two frequency excitation at various choices of the fundamental frequency f and its subharmonic f/2 with controlled initial phase difference ϕ_{in} between them. We explore

the effects of the controlling parameters, namely: (i) forcing frequencies and their initial amplitudes, (ii) initial phase difference ϕ_{in}, and (iii) detuning (i.e. when the second forcing frequency is slightly different from f/2). While several of our experimental observations support predictions based on weakly nonlinear theory, others do not. We explain our data in terms of vortex dynamics concepts.

Paper 22

Husain, H.S. & Hussain, F., "Experiments on subharmonic resonance in a shear layer." *J. Fluid Mech.* (in press).

20. Spatiotemporal Dynamics in Open Flows

We have developed an approach combining dynamical systems analyses with spatial measures — coherence and cross-bicoherence (proposed as measures of spatial coupling) — to identify and describe low-dimensional *global dynamics* in an *open* flow. To demonstrate this approach, experiments were performed in an initially laminar, periodically forced plane mixing layer in a low-noise, anechoic environment. We used dynamical invariant measures, with forcing frequency and amplitude as control parameters, to show that the transitional mixing layer dynamics of vortex roll-up and the first two pairings can be described by one periodic and at least two low-dimensional chaotic attractors. A phase diagram delineating the dynamical states associated with these attractors was constructed. The spatial extents of these states (quantified by coherence and cross-bicoherence measurements) span many instability wavelengths downstream, showing that the associated flow states are *global.* At a fixed forcing frequency, as the forcing amplitude was decreased, an intermittency transition occurred from the periodic attractor to a chaotic attractor. Following this transition, the second pairing dynamics, which was shown to be spatially coupled for the periodic attractor, became chaotic and spatiotemporal. This spatiotemporal dynamics, inferred from decay of coherence and cross-bicoherence, was accompanied by a sudden increase in the attractor dimension. Loss of spatial coupling during the chaotic second pairing, *i.e.* disrupted global dynamics, led to what appeared to be *spatiotemporal chaos.* This approach of combining temporal analyses and spatial measures seems promising for studying spatiotemporal dynamics in other open shear flows.

Paper 23

Narayanan, S. & Hussain, F., "Measurements of spatiotemporal dynamics in a forced plane mixing layer." *J. Fluid Mech.*, 1995 (submitted).

21. Structural Model of a Turbulent Mixing Layer

We have combined a structural approach, by deriving a turbulent structure, which we call an *eigenlet*, as an eigenfunction of the Navier-Stokes equations, with turbulence modeling using a new curl-type eddy viscosity model, to describe a fully developed turbulent mixing layer without using any empirical input. This result was achieved by invoking the self-consistency condition that the mean flow and the eigenlet have the same spread angle. Using self-similar variables and the modeled equations, we obtained the mean flow and eigenlet. Furthermore, several flow features such as Reynolds stress, mixing layer spread and inclination angles, and the average structure passage frequency were calculated; these results are in good agreement with experimental data.

<u>Paper 24</u>

Goldshtik, M. & Hussain, F., "Structural approach to the Modeling of a Turbulent Mixing Layer." *Phys. Rev. E*, 1995 (in press).

22. Dynamics of Point Vortices in Bounded and Unbounded Domains

Motion of point vortices are relevant to understanding stirring of passive fluid particles in inviscid, incompressible flows. We have investigated the dynamics of a pair of point vortices of opposite signs in a rectangular domain for the whole range of energies. This simplest nonintegrable system reflects important features of different vortex flows: Benard convection, Gortler vortices, flow in a cavity etc. Instead of using approximate cloud-in-cell method for many vortices, we derived the exact equations of motion for an arbitrary system of point vortices in a rectangle and applied it to the particular case of a pair of vortices. Patterns of periodic, quasiperiodic, chaotic and billiards-type motions are revealed for generic cases and two limiting cases: two close vortices (near-dipole) and a vortex close to the boundary. We also introduce new criteria of ergodicity and find that some chaotic motions are close to, in some sense, ergodic ones. Although the motion of a point vortex in a bounded domain is integrable and thus periodic, it has been conjectured that the corresponding motion of fluid particles in the Lagrangian representation (advection) is generally chaotic and leads to stirring of fluid. Thus, we are investigating chaotic advection induced by a family of periodic motions of a point vortex inside or outside of a circular domain and also in an infinite plane. Though very simple, these cases provide a good qualitative picture of regular and chaotic advection for arbitrary domains.

Paper 25

Kunin, I.A., Hussain, F., & Zhou, X., "Dynamics of a pair of vortices in a rectangle." *Lett. App. & Engr. Sci.,* **32**, pp1835-1844, (1994).

23. Routes to Chaos in Advection Induced by Point Vortices

Chaotic advection induced by periodic motions of point vortices inside and outside of a circular domain and in the infinite plane were analyzed. Though simple, this model flow captures multiple routes to chaos and gives a good qualitative picture of advection in more complicated domains such as rectangle and ellipse. Different scenarios of chaotic advection depend on the topology of unperturbed phase portraits and type of perturbations. In addition to the typical KAM cases, we found scenarios accounting for inherent singularity of the velocity field.

Paper 26

Kunin, I.A., Hussain, F. & Zhou, X., "On multiple routes to chaos in advection induced by point vortices." *Phys. Rev. E.* 1995 (submitted).

Paper 27

Kunin, I. A., Zhou, X., "Chaotic advection by point vortices I. Infinite and circular domains." 1995 (in preparation).

Paper 28

Kunin, I. A. , Zhou, X. , " Chaotic advection by point vortices II. Arbitrary domains." 1995 (in preparation).

REVIEW PAPERS

Paper 29

Hussain, F., Virk, D. & Melander, M.V. "New studies in vortex dynamics: incompressible and compressible vortex reconnection, core dynamics, and coupling between large and small scales." *Acad. Proc. in Engr. Sci. (Indian Acad. of Sci.),* **18**, pp477-529, (1993).

Paper 30

Schoppa, W. & Hussain, F., "New aspects of vortex dynamics relevant to coherent structures in turbulent flows." *Euromech Symp. Vortex Dynamics*, Cortona, Italy (1993).

Paper 31

Virk, D. , Schoppa, W. & Hussain, F. , "DNS studies of vortex dynamics of relevance to turbulent flows." *CFD Review* (eds. M. Hafez & K. Oshima), J. Wiley, pp. 641- 678 (1995).

SOME INVITED LECTURES

1) " Understanding turbulence via vortex dynamics: some new perspectives." *5th Asian Congress of Fluid Mechanics*, Taejon, Korea, Aug. 10-14 (1992).

2) " Internal instability of divergent flows."*18th ICTAM*, Haifa, Israel, Aug. 22-28 (1992).

3)" New perspectives on vortex dynamics: Core dynamics, Helical Wave Decomposition and interaction with turbulence." *7th Beer-Sheva Intnl. Sem.. on MHD & Turb.*, Jerusalem, Israel, Feb. 14-18 (1993).

4) " Subharmonic resonance in free shear layers. " *IUTAM Symp. on Nonlinear Instabilities of Nonparallel Flows*. Clarkson U., Jul. 25-30 (1993).

5) " A new mechanism for transition in free shear layers: vortex core dynamics. " *Symp. on Developments in Fluid Dyn. & Turb.*, Bangalore, India, Dec. 9-10 (1993).

6) " Topological fluid mechanics and vortex reconnection." *Turb. as a Problem in Physics*, Nehru Center for Advanced Scientific Research, Bangalore, Dec. 13-18, (1993).

7) " Core dynamics instability: A new transition mechanism in a mixing layer. " *Advances in Turbulence Research -1995*, Pohang, Korea, Mar 27-29, (1995).

8) " Vortex dynamics and Turbulence." *Tani Memorial Lecture*, 6th Asian Congress of Fluid Mechanics, Singapore, May 22-26, (1995).

CONFERENCE PRESENTATIONS

1) Shtern, V.N., Goldshtik, M. & Hussain, F. "Hysteretic vortex breakdown." *Bull. Am. Phys. Soc.*, **37**, p1707, (1992).

2) Goldshtik, M. & Hussain, F. "Stagnation zone theory and vortex breakdown." *Bull. Am. Phys. Soc.*, **37**, p1707, (1992).

3) Schoppa, W., Metcalfe, R.W. & Hussain, F. "The helical pairing instability - An alternative 3D evolution of a plane mixing layer." *Bull. Am. Phys. Soc.*, **37**, p1801, (1992).

4) Goldshtik, M., Shtern, V.N. & Hussain, F. "Rudiments of a theory of structural turbulence." *Bull. Am. Phys. Soc.*, **38**, p2296, (1993).

5) Shtern, V.N. & Hussain, F. "Jump transitions in swirling jets." *Bull. Am. Phys. Soc.*, **39**, p1972, (1994).

6) Goldshtik, M. & Hussain, F. "Some intriguing features of axisymmetric jets." *Bull. Am. Phys. Soc.*, **39**, p1950, (1994).

7) Narayanan, S. & Hussain, F. "Spatiotemporal dynamics in a forced plane mixing layer." *Bull. Am. Phys. Soc.*, **39**, p1843, (1994).

8) Metcalfe, R.W., Schoppa, W., Park, K. & Hussain, F. "The potential for mixing enhancement in a free shear layer through core dynamics instability." *Bull. Am. Phys. Soc.*, **39**, p1895, (1994).

9) Narayanan, S. & Hussain, F., " Spatiotemporal dynamics in a plane mixing layer. " *6th Asian Congress of Fluid Mechanics*, Singapore, May 22-26, (1995).

10) Schoppa, W., Hussain, F. & Metcalfe, R. W., " A new mechanism of small scale transition in a plane mixing layer : core dynamics of spanwise vortices." *6th Asian Congress of Fluid Mechanics*, Singapore, May 22-26, (1995).